Modelling of tension in yarn package unwinding

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ABSTRACT – REZUMAT

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Yarn unwinding from a package is an essential step in many textile processes. The quality of the yarn is numerically expressed mainly by values of mechanical quantities. In the unwinding process viscoelastic properties are the most important ones. They depend on how the yarn is stressed. The quality of the yarn that is being unwound should not be reduced, unless this reduction doesn't significantly lower the quality of the textile fabric. During unwind the yarn tension is not constant, but it oscillates within some interval. Even when the yarn is not strongly stressed, the yarn still can break sometimes. This is why we think that a cross-wound package is not an ideal form of a package and that such packages aren't always made without flaws. We strive to achieve as large warping and weaving speeds as possible, therefore our aim is to improve the theory of cross-wound package unwinding and to find the necessary modifications of the yarn unwinding process.

Keywords: *modelling, tension, dynamics of yarn, balloon theory, quasi-stationary approximation*

Modelarea tensiunii în derularea bobinei de fire

Derularea firelor dintr-o bobină este un pas esențial în multe procese textile. Calitatea firului este exprimată numeric în principal prin valori ale mărimilor mecanice. În procesul de derulare proprietățile viscoelastice sunt cele mai importante. Acestea depind de modul în care firul este solicitat. Calitatea firelor care se derulează nu trebuie redusă, cu excepția cazului în care această reducere nu scade semnificativ calitatea materialului textil. În timpul derulării, tensiunea firului nu este constantă, dar oscilează într-un anumit interval. Chiar și atunci când firul nu este solicitat puternic, acesta se poate rupe uneori. Acesta este motivul pentru care se consideră că o bobină încrucișată nu este o formă ideală și că astfel de bobine nu sunt întotdeauna realizate fără defecte. Se dorește obținerea de viteze cât mai mari de urzire și țesere, prin urmare, scopul este să se îmbunătățească teoria derulării bobinei încrucișate și să se identifice modificările necesare procesului de derulare al firului.

Cuvinte-cheie*: modelare, tensiune, dinamica firelor, teoria derulării firului, aproximare cvasi-staționară*

INTRODUCTION

The theory of yarn unwinding and the balloon theory has a long history. Different authors tried to develop it and influenced the theory through the certain period of time [1–7]. The theory as we know it today was heavily influenced by Fraser, Ghosh and Batra [8]. They applied perturbation theory to show how to eliminate time dependence from the equation of motion of a cylindrical packages in a mathematically correct way. They derived moving boundary condition for packages with small winding angles. Fraser also found out that for elastic yarn the tension in the balloon is smaller [9, 10]. However, it turns out that this effect is small for elastic constants encountered in typical yarns [11]. The theory of yarn movement on the surface of the package was developed simultaneously with the balloon theory. Both theories solved the simplified equations at stationary boundary conditions and so determined the length of the sliding yarn [3]. The computation was verified by Fraser et al. [9]. During unwinding from the package the yarn moves over the surface of the package. The point where the yarn leaves the package is called a lift-off point. The residual tension of the yarn from the interior of the package is released at this point. The equations of motion of the yarn are known. We derived them in our previous contribution [12]. As we will show here it is possible to obtain a partial analytical solution demonstrating the existence of the residual force.

GENERAL EQUATION OF MOTION FOR YARN

The yarn is unwinding from a fixed cylindrical package in horizontal direction with velocity V through the guide-eye O (figure 1). The origin of the coordinate system is at the guide-eye. Point Lp is the lift-off point, i.e., the point where the yarn leaves the surface of the packages to form the balloon. Angle ϕ is the angle of the winding on the package. We are interested in balloon motion, i.e., the time variation of the position radius vector $r(s,t)$ of the yarn in space.

The theory of yarn unwinding off a package and the balloon equations was derived in the previous work [12]:

$$\rho\left(D^2 r + 2\omega \times Dr + \omega \times (\omega \times r) + \dot{\omega} \times r\right) = \frac{\partial}{\partial s}\left(T\frac{\partial r}{\partial s}\right) + f \quad (1)$$

They can be partially analytically solved, as we show in the following. Fictitious forces on the left-hand side of the equation are: the Coriolis force $-\rho 2\omega\times Dr$, the centrifugal force $-\rho\omega\times(\omega\times r)$ and the Euler force $-\rho\dot{\omega}\times r$. D is the differential operator which follows the motion of a point on the yarn in the rotating reference frame [8]:

$$D = \frac{\partial}{\partial t}\bigg|_{r,\theta,z} - V\frac{\partial}{\partial s} \quad (2)$$

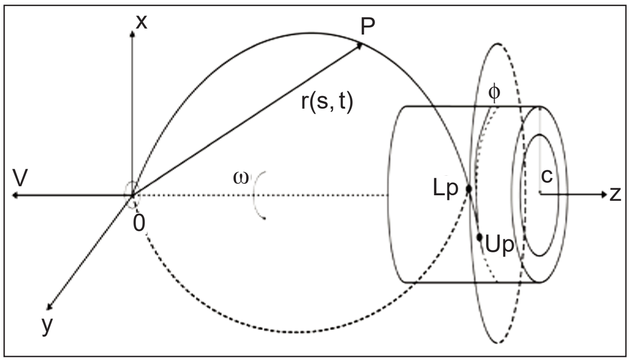

Fig. 1. Yarn unwinding from a cylindrical package

The fact that this operator "follows the motion of the point in the rotating frame" means, that the partial time derivative operator only operates on the coordinates of the point (r, θ, z), but it gives zero when applied on the base vectors e_z, e_θ, e_r. T is the mechanical tension and f is the linear density of external forces. In yarn which forms the balloon, f is the air resistance force [13, 14]:

$$f = -\frac{1}{2} c_u \rho d |v_n| v_n \quad (3)$$

where c_u is the effective air-drag coefficient, ρ – the mass per unit length of the yarn, d – the yarn diameter and v_n – the normal component of the yarn velocity. With the help of $D_n = 1/2 c_u \rho d$ we can write the air resistance force in a similar form as found in literature [12]:

$$f = -D_n |v_n| v_n \quad (4)$$

The equation of motion 1 expressed in the dimensionless form. We express all distances in units of package radius $\bar{r} = r/c$, $\bar{r} = r/c$, $\bar{z} = z/c$, $\bar{s} = s/c$, time is expressed in units of period of balloon rotation: $\bar{t} = t/\tau = \omega t$, velocitis are expressed in units of unwinding speed: $\bar{v} = v/V$, $\bar{v}_n = v_n/V$ and finally we find the following suitable combinations of quantities for forces and tension [15]:

$$\bar{f} = \frac{fc}{\rho V^2}, \bar{n} = \frac{nc}{\rho V^2}, \bar{T} = \frac{T}{\rho V^2} \quad (5)$$

Transforming the equantion of motion into the dimensionless form we get:

$$\bar{D}^2\bar{r} + 2\Omega\times\bar{D}\bar{r} + \Omega\times(\Omega\times\bar{r}) + \Omega\frac{\partial\Omega}{\partial\bar{t}}\times\bar{r} = \frac{\partial}{\partial\bar{s}}\left(\bar{T}\frac{\partial\bar{r}}{\partial\bar{s}}\right) + \bar{f} \quad (6)$$

where:

$$\bar{D} = \Omega\frac{\partial}{\partial\bar{t}} - \frac{\partial}{\partial\bar{s}}, \ \Omega = \frac{c\omega}{V} \quad (7)$$

The dimensionless air resistance force is:

$$\bar{f} = -\frac{p_0}{16}|\bar{v}_n|\bar{v}_n \quad (8)$$

where:

$$\bar{v} = \bar{D}\bar{r} + \Omega\times\bar{r} \quad (9)$$

and

$$\bar{v}_n = \frac{\partial\bar{r}}{\partial\bar{s}}\times\left(\bar{v}\times\frac{\partial\bar{r}}{\partial\bar{s}}\right) \quad (10)$$

QUASI-STATIONARY APPROXIMATION

So far our derivation was entirely general. We took into account that the yarn has constant linear mass density and that it is unstretchable. The dimensionless equation 6 derived above enables to study arbitrary yarn motion. In the literature we can find two different approaches. In the first approach we simply assume that the motion is quasi-stationary with respect to the rotating coordinate system by setting all time derivatives on zero. In the second approach we attack the problem with perturbation theory [8]. Namely, it turns out that with a suitable choice of dimensionless variables the motion equations can be written in a form where all time derivatives are multiplied with a small parameter. Fraser estimates that for a typical package this parameter is between 0.007 and 0.103, which shows that time derivatives can be neglected in the first approximation. Perturbation theory is a systematic approach for founding such approximations. Fraser shows that the fundamental equation (the equation in the so called zero order) describes stationary motion et yarn in the rotating coordinate system. Boundary conditions however can be time dependent. Besides that it turns out that $\Omega = 1$ in the zero order of the theory. This corresponds to the winding angle $\phi = 0°$, which limits the generality of solutions if we work in the fundamental (zero) order perturbation theory as Fraser did. Unfortunately equations for corrections are time dependent which is not very helpful. The use of perturbation theory is more justified from a mathematical point of view but from the perspective of physics the first approach is also entirety satisfactory and it also allows greater generality. Therefore we have decided to use quasi-stationary approximation in the sequel. If we sent all time derivatives to zero and we omit writing ¯ for dimensionless quantities we get the following quasi-stationary equation of motion.

$$\frac{\partial^2 r}{\partial s^2} - 2\Omega\times\frac{\partial r}{\partial s} + \Omega\times(\Omega\times r) = \frac{\partial}{\partial s}\left(T\frac{\partial r}{\partial s}\right) + f \quad (11)$$

A DERIVATION OF COMPONENTE WISE EQUATIONS OF MOTION

The vectorial equation 11 will be written out in component form. This form is more suitable for solving the equations.
Firstly, we need the first and the second derivative of the position vector.

$$r(r,\theta,z) = r(s)\,e_r(\theta(s)) + z(s)\,e_z \quad (12)$$

We emphasize that the basis vector e_r depends on the angle θ. Therefore it indirectly depends also on the parameter s. To compute derivatives we use relations [12]:

$$\frac{\partial e_r}{\partial\theta} = e_\theta \qquad \frac{\partial e_\theta}{\partial\theta} = -e_r \quad (13)$$

For the first derivative we get that:

$$\frac{\partial r}{\partial s} = r'e_r + r\frac{\partial e_r}{\partial\theta}\frac{\partial\theta}{\partial s} + z'e_z = r'e_r + r\theta' e_\theta + z'e_z \quad (14)$$

We introduced notation $' = \partial/\partial s$ for derivatives with respect to parameter s.
For the second derivative we get that:

$$\frac{\partial^2 r}{\partial s^2} = r''e_r + r'\frac{\partial e_r}{\partial\theta}\frac{\partial\theta}{\partial s} + r'\theta'e_\theta + r'\theta''e_\theta + r\theta'\frac{\partial e_\theta}{\partial\theta}\frac{\partial\theta}{\partial s} + z''e_z$$
$$= r''e_r + r'\theta'e_\theta + r'\theta'e_\theta + r\theta''e_\theta - r'\theta'\theta' e_r + z''e_z$$
$$= (r'' - r\theta'^2)e_r + 2r'\theta'e_\theta + r\theta''e_\theta + z''e_z \quad (15)$$

To make bottle results more transparent we write them as column vector:

$$r' = \begin{bmatrix} r' \\ r\theta' \\ z' \end{bmatrix} \qquad r'' = \begin{bmatrix} r'' - r\theta'^2 \\ 2r'\theta' + r\theta'' \\ z'' \end{bmatrix} \quad (16)$$

We will also need the following results which are obtained from the rules for computing with vector products:

$$\Omega\times r = \Omega\begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix},\ \Omega\times r' = \Omega\begin{bmatrix} -r\theta' \\ r' \\ 0 \end{bmatrix},\ \Omega\times(\Omega\times\bar{r}) = \Omega^2\begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix} \quad (17)$$

Considerably more work is required for the computation of the normal component of velocity. We will need this in expression for the density of the air resistance force. We start with the velocity which is by equation 9 equal to:

$$v = -r' + \Omega\times r = \begin{bmatrix} -r' \\ -r\theta' + \Omega r \\ -z' \end{bmatrix} \quad (18)$$

The normal component of velocity can be written as:

$$v_n = r'\times(v\times r') = \Omega\begin{bmatrix} -r^2\theta' r' \\ rz'^2 + rr'^2 \\ -r^2\theta' z' \end{bmatrix} = \Omega r\begin{bmatrix} -r\theta' r' \\ z'^2 + r'^2 \\ -r\theta' z' \end{bmatrix} \quad (19)$$

The square of the norm of this vector is:

$$|v_n|^2 = \Omega^2\left((r^2\theta' r')^2 + r^2(z'^2 + r'^2)^2 + (r^2\theta' z')^2\right)$$
$$= \Omega^2 r^2\left(r^2\theta'^2 r'^2 + z'^4 + 2z'^2 r'^2 + r'^4 + z'^2 r^2\theta'^2\right)$$
$$= \Omega^2 r^2\left(r'^2(\theta'^2 r^2 + z'^2 + r'^2) + z'^2(\theta'^2 r^2 + z'^2 + r'^2)\right)$$
$$= \Omega^2 r^2 (r'^2 + z'^2)(r'^2 + r^2\theta'^2 + z'^2)$$
$$= \Omega^2 r^2 (r'^2 + z'^2) \quad (20)$$

In the last step we used the inextensibility condition, $r'^2 + r^2\theta'^2 + z'^2 = 1$. Therefore the normal component of velocity is:

$$v_n = \Omega r\sqrt{r'^2 + z'^2} \quad (21)$$

It follows that the linear density of air resistance is equal to:

$$f_u = -\frac{1}{16}\,p_0 v_n v_n$$
$$= -\frac{1}{16}\,p_0 v_n \Omega\begin{bmatrix} -r^2 r'\theta' \\ r(r'^2 + z'^2) \\ -r^2\theta' z' \end{bmatrix} = \begin{bmatrix} 1/16\,p_0\,\Omega r^2 r'\theta' v_n \\ -1/16\,p_0/(\Omega r)\,v_n^3 \\ 1/16\,p_0\,\Omega r^2\theta' z' v_n \end{bmatrix} =$$
$$= -\frac{1}{16}\begin{bmatrix} p_0\,\Omega r^2 r'\theta' v_n \\ -p_0 v_n^3/(\Omega r) \\ p_0\,\Omega r^2\theta' z' v_n \end{bmatrix} \quad (22)$$

By using the formula $\partial/\partial s\,(T\partial r/\partial s) = T'r' + Tr''$ we can write the equation of motion as

$$(1 - T)r'' - 2\Omega\times r' + \Omega\times(\Omega\times r) = T'r' + f \quad (23)$$

This is a vectorial equation with three components:

$$(1 - T)(r'' - r\theta'^2) + 2\Omega r\theta' - \Omega^2 r = T'r' + \frac{1}{16}\,\Omega p_0 r^2 r'\theta' v_n \quad (24)$$

$$(1 - T)(2r'\theta' + r\theta'') - 2\Omega r' = T'r\theta' - \frac{1}{16}\,p_0 v_n^3/(r\Omega) \quad (25)$$

$$(1 - T)z'' = T'z' + \frac{1}{16}\,\Omega p_0 r^2\theta' z' v_n \quad (26)$$

The fourth equation is the inextensibility condition:

$$r'^2 + r^2\theta'^2 + z'^2 = 1 \quad (27)$$

Therefore we have four equations for four variables, r, θ, z in T. We multiply the equation (24) with r', the equation (25) with $r\theta'$ and the equation (26) with z' to get:

$$(1 - T)\left(r''r' + r'r\theta'^2 + r^2\theta'\theta'' + z'z''\right) - rr'\Omega^2 =$$
$$= T'(r'^2 + r^2\theta'^2 + z'^2) +$$
$$+ \frac{1}{16}\,p_0 v_n\Omega\left(r^2 r'^2\theta^2 - \theta' v_n^2/\Omega^2 + r^2\theta' z'^2\right) \quad (28)$$

The expression between the round brackets on the right-hand side of the equation is equal to 1, by the inextensibility condition. The expressions between the square brackets are both equal to 0. The proof for the first expression is given by the following equation:

$$(r'r'') + (z'z'') + (r'r\theta'^2 + r^2\theta'\theta'') =$$
$$= \left[\frac{1}{2}(r'^2) + \frac{1}{2}(z'^2) + \frac{1}{2}(r^2\theta'^2)'\right]'$$
$$= \frac{1}{2}(r'^2 + z'^2 + r^2\theta'^2)'$$
$$= \frac{1}{2}(1)' = 0 \quad (29)$$

where we used the inextensibility condition in the next to last line.
In the second expression we first expand v_n^2 by using equation 21, we see that all terms cancel out. Therefore equation 28 simplifies to:

$$T' = -\Omega^2 rr' \quad (30)$$

If we write T_0 for the tension in the yarn passing through the guide (figure 1) we get that:

$$T = T_0 - \frac{1}{2}\Omega^2 r^2 \quad (31)$$

PARTIAL ANALYTICAL SOLUTION

When the yarn is unwinding from the package the lift-off point moves over a two-dimensional surface. We can use this in equation 31 to obtain another two-dimensional problem which can be solved more easily. It turns out that the tension of the yarn in the interior of the package can be expressed analytically. We will therefore assume that our package has a cylindrical shape (figure 2), which implies that the radius-vector to a surface point of the package is [12]:

$$r = ce_r + ze_z \quad (32)$$

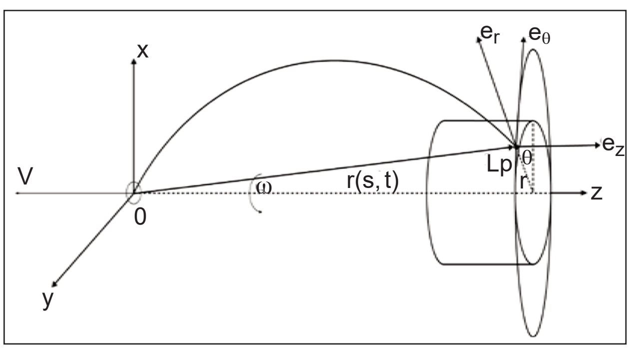

Fig. 2. Coordinate frame correspond to equation (31)

The quantity c is by definition equal to the constant distance of the point r from the axis of the packag, it is though equal to the diameter of the layer which is currently unwinding [12]. The eqaution 31 will now be used to find the residual tension at the lift-off point Lp. Therefore we are interested in the velocity with which the yarns leaves the package. This velocity doesn't have to be equal to the unwinding velocity V_1. Namely we have that

$$V_1 = V + \dot{s}_1 \quad (33)$$

where s_1 is the lenght of the yarn in the balloon i.e. the lenght of the yarn between the guide and the lift-off point Lp. In other woords we had to add time derivative of lenght to the velocity V to obtain the lift-off velocity. We take into account that:

$$\frac{ds_1}{dt} = \frac{ds_1}{dz_1}\frac{dz_1}{dt} \quad (34)$$

where z_1 is the z coordinate of the lift-off point.
At quasistationary movement we have that:

$$\frac{ds_1}{dz_1} = 1 \quad (35)$$

because the length of the yarn between the package and the guide is enlarged exactly by the enght corresponding to the displacement of the point Lp. If we insert the condition 35 for quasistationary movement into equation 32 we obtain boundary conditions at the lift-off point which we can express as $r = c$. The dimensionless boundary condition at the lift-off point becomes:

$$r(s_{Lp}, t) = 1 \quad (36)$$

Inserting condition (35) into equation 31 we get that the tension at the Lp is then equal to:

$$T - T_0 = \frac{1}{2}\Omega^2 \quad (37)$$

The equation 37 tells us that tension in the yarn drops from its value in the balloon (at the lift-off point) to its residual value, defined as the tension of the yarn inside the package. If we write $T_R = \Omega^2/2$ for the residual tension in the yarn (figure 3), we get that:

$$T - T_0 = T_R \quad (38)$$

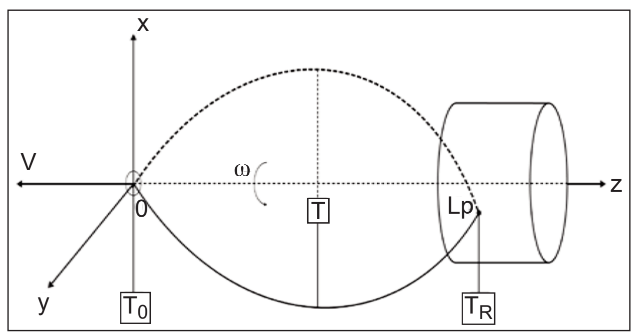

Fig. 3. View of the "balloon" (illustration of equations 38)

CONCLUSIONS

We saw how the equation for yarn tension in the balloon can be obtained from the general equation of yarn by replacing the usual perturbation theory approach with quasi-stationary approximation. The tension T of the yarn in the balloon consists of two points: from the tension T_0 in the yarn at the guide-eye and from the residual tension T_R in the yarn: $T - T_0 = T_R$. Analytical solutions enable a better understanding of interdependencies between various quantities. The residual tension in the yarn has not been studied enough so far. We showed that it enables a reduction of the yarn tension in the balloon. This is the first analytical proof of the existence of the residual force in the theory of yarn unwinding.

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